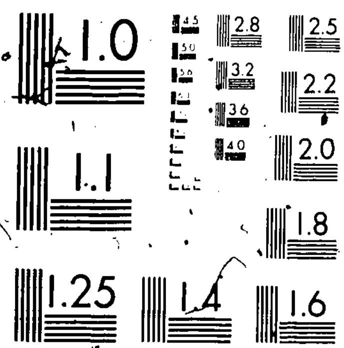

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ABSTRACT

Four procedures are stated which are inherent in problem solving: (1) understanding the problem; (2) planning to solve the problem; (3) solving the problem; and (4) reviewing the problem and the solution. Each of the four procedures is illustrated using problems appropriate for elementary school children. These illustrations are accompanied by some guidelines and instructional moves which can be used to help children in their problem solving. Two different problems are used: (1) a typical textbook problem; and (2) a process problem. (MP)

MATHEMATICAL PROBLEM SOLVING PROJECT

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MATHEMATICS EDUCATION DEVELOPMENT CENTER

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FINAL REPORT
MATHEMATICAL PROBLEM SOLVING PROJECT
TECHNICAL REPORT I:
DOCUMENTS RELATED TO A PROBLEM-SOLVING MODEL
PART C: "You Can Teach Problem Solving!"

Report Prepared By

John F. LeBlanc
Project Director

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You Can Teach Problem Solving!

Teaching problem solving is a problem! But like most other problems it can be solved. And like other teaching situations there is no formula that can be automatically applied to guarantee success in teaching problem solving. But there are some guidelines and hints which can go a long way in aiding a teacher in problem-solving instruction.

Before these guidelines and hints are explored a few general comments might be helpful to place the focus of this article in context. There are three recognized aspects of teaching mathematics in the school: computational proficiency, concept attainment, and problem solving. (Some mathematics educators state that problem solving is the goal of school mathematics!) Each aspect deserves appropriate emphasis and requires a different instructional approach. The instructional approach to be used depends on the goals to be attained. In teaching problem solving the instructional goal should be to help children learn procedures for solving problems. These procedures are easy to state and recognize but they are often quite elusive when teaching. It seems so much easier to help children acquire a computation skill or to help them understand a concept than to help them acquire skill with problem-solving procedures. Among the reasons for the difficulty in teaching/learning problem-solving procedures is that there is no specific content as in computational skills or in concept attainment. In problem solving an individually acquired set of processes is brought to bear on a situation which confronts the individual. Helping the child acquire and apply these processes is a more complicated and less well-defined task than to help a child learn a computational skill or understand a concept.

What are the procedures which are inherent in problem solving? There are four procedures that are stated in various ways by a number of authors. They are:

1. Understanding the problem
2. Planning to solve the problem
3. Solving the problem
4. Reviewing the problem and the solution

How can a teacher help children acquire and use the four procedures? In the discussion which follows, each of the four procedures will be illustrated using problems appropriate for elementary school children. These illustrations will be accompanied by some guidelines and instructional moves which can be used to help children in their problem solving. Two different problems will be used in the discussion. The first is a typical textbook problem. The second problem can be called a "process" problem.

The basic purpose of the typical textbook problem is to reinforce children's understanding of a concept or to use a skill learned earlier by presenting a "real-world" situation which embodies that concept or skill. Using these typical textbook problems to teach problem-solving procedures has the shortcoming that for many children the solution or at least the plan for solving it is so apparent that no problem exists. However, a teacher can emphasize the procedures for the children.

Example 1

A shopkeeper sold 485 bottles of cola in one day. The cola is packaged in cartons holding 6 bottles. How many cartons of cola were sold?

1. Understanding the problem

In helping children understand the problem a teacher might use questions which focus the child's attention on the information or conditions given in the problem.

"John, how many bottles were sold?"

"Nina, how are the cola bottles packaged?"

"Felix, how many bottles in a carton?"

"What does a carton look like?"

"Does everyone always buy a full carton? Does that make any difference in our problem?"

"If one carton were sold, how many bottles of cola would be sold?"

Finally, a teacher might ask,

"What does the problem ask for? Can you tell it in your own words?"
Until a thorough discussion of the information and conditions takes place, this last question might make little sense. Certainly, advice such as "Think!" and "Read the problem again!" does not help the child without accompanying questions which focus on understanding the problem.

In the above sequence of questions the teacher has helped the children learn to help themselves understand a problem. Teachers should call upon children to do the question asking, and to formulate questions for their classmates. This experience will help them grow in their ability to focus on important information.

2. Planning to solve the problem

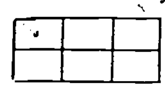
In the discussion of this procedure a teacher should be aware of and look for different strategies which might be used in solving the problem. Initially, the teacher may have to suggest possible strategies for the children. After some experience using various strategies children will choose their own. Throughout the planning-to-solve-the-problem step, the teacher should emphasize the type of strategy the child has suggested.

"Kids, how might we plan to solve this?"

1st student: "Well, we could get 485 bottles or chips and put them into groups of 6."

"Good suggestion. In other words, we could actually conduct an experiment."

2nd student: "It helps me to draw something like a carton so I can think about six bottles in each."



"I like that strategy. Making a drawing or a diagram is a good way to visualize the problem."

3rd student: "Well it takes 6 bottles for one carton and so for 10 cartons 60 bottles would be used. I would add 60's until I got to 485 or near it."

"Good thinking! Using patterns is an excellent way of solving some problems. You could even put your numbers into a table."

Cartons	10	20	30	etc.
Bottles	60	120	180	

4th student: "I would divide 485 by 6 to find the number of cartons."

"Good. Sometimes direct computation is a good strategy to try."

One of the shortcomings of the standard textbook problems (as was mentioned earlier) is that they are often designed to provide computational practice rather than to teach problem solving. This places an additional burden on the teacher to create a problem-solving situation by suggesting or trying to elicit other strategies.

It was mentioned above that such suggestions should come from the students but initially the teacher must supply some of the strategies*.

* Strategies may be grouped into general and helping. A general strategy may be thought of as an overall plan designed to solve the problem. A helping strategy may be thought of as an intermediate step used to carry out the general strategy. Some general strategies are: trial and error, organized listing, simplification, searching for a pattern, experimentation, deduction, computation, working backwards. Some helping strategies are: diagrams, tables, graphs, lists, equations.

Also note that for some students complete understanding of the problem may come at this step rather than in the first step.

3. Solving the problem

In this step the plan selected in step 2 is carried out. The teacher should encourage the children to solve the problem using the variety of plans elicited earlier. The importance of this step is not how efficiently the problem can be solved. Rather this step should be viewed as the step that carries out the plan. For some children the plan selected will not lend itself to solution (e.g., diagramming n cartons of 6 until 485 bottles are drawn!). At this point the problem solver should return to step 2 to devise another plan.

The teacher's role in promoting this step should be advisory in that he/she should ask how the solution process fits the selected plan. At this point there are temptations which afflict teachers. There is an impulse to overemphasize the "solving-the-problem" step, thereby minimizing the other steps. Secondly, there is a strong tendency to so press for computational accuracy that problem solving is associated in the minds of children only with onerous computation.

4. Reviewing the problem and its solution

In developing problem-solving procedures in children there is no step more important than this fourth step. There are two aspects of this step; one is looking back over the steps taken and the other is extending the problem situation to create variations or an entirely new problem. The former aspect should never be omitted in any problem-solving instruction.

"Tell us how you thought about solving it."

"Show us at the board how you worked it out."

"What strategy did you use to get your answer?"

"Does this solution fit the question in the problem?"

"Does that answer make sense to you?"

All of the above comments are asking the children to introspect, to look at their own thinking and describe it. This activity is perhaps the most important part of the instruction procedure. It benefits the child doing the introspection and it benefits the other children who hear how someone else thinks about the problem.

In extending the problem, the teacher could ask,

"Suppose the shopkeeper puts 4 cartons into each case. How many cases would he fill?"

"What if there were 600 bottles; could we solve the problem using the same strategies?"

"What profit did the shopkeeper make if he made 4 cents (4.3 cents!) on each bottle?"

The typical textbook problem is not an ideal vehicle for teaching problem-solving procedures since most of them by design can be solved by direct application of an arithmetic algorithm (addition, division, percent, averaging, etc.). Further, most of the typical textbook problems follow a section of skill or concept development and are related to that skill or some applications just developed. In such a situation it takes little "problem-solving" ability to guess an appropriate strategy! On the other hand it is important to use every opportunity possible to teach problem-solving procedures, even using the standard textbook problem.

There is another problem type, a "process" problem, which is so called because it lends itself to exemplifying the procedures inherent in problem solving. Process problems are found in several textbooks in the corners of some pages. While these problems are presently often viewed as enrichment or as recreational, they are fast becoming recognized as an important vehicle for teaching problem solving. In the near future they may become part of the mainstream elementary mathematics curriculum. Not only can process problems be solved using a variety of strategies but they also require very little formal mathematics. The second example, which follows, is a process problem.

Example 2

"There are 8 people at a party. If each person shakes hands with the other guests, how many handshakes will there be?"

1. Understanding the problem

As in the previous example, a teacher can help the children understand the problem by selected questions. Again, the children should be encouraged to ask the questions themselves.

"How many guests are at the party?"

"If Jim shakes hands with Carole, should Carole later shake hands with Jim?"

"If you were at the party, how many people would you shake hands with?"

"Now who can give the problem in his/her own words?"

2. Planning to solve the problem

One of the features of the "process" problem is that it lends itself to many different strategies for solution. The following are some of the strategies which children came up with in regular fourth- and fifth-grade classrooms.

1st student: "Let's act it out by using 8 kids. Come on, let's get in a circle!" (Experiment)

2nd student: "I'm going to draw it out by putting 8 dots in a circle and using lines to show handshakes." (Diagram)

3rd student: "I'm going to give them names and make a list of the people each shakes hands with." (Organized Listing)

4th student: "I wonder if there is a pattern. Let's see, if there are 2 people, there is 1 handshake; if there are 3 people there are 2 handshakes. . ." (Using tables to find a pattern)

People	2	3	4	5 . . .
Handshakes	1	3	6	

5th student: "Well, if each person shakes hands with the others, then 8 people shake hands with 7 others or there are 8 x 7 handshakes. But..." (Deduction)

The above five responses came from children after several problems had been solved together with their teacher. The teacher may (but not always) have to initially supply some of the strategies. He/she will have to identify and label the strategies of the children so they will learn to use the labels in communicating to others.

3. Solving the problem

The children usually do not have any difficulty solving the problem once a strategy has been selected. Even if the strategy is inappropriate, the insight acquired by following the inappropriate strategy usually helps the child to select another strategy.

In the classroom setting, solution of the problems brings enough questions from other children to forestall the apparent error in the

fifth child's deductive thinking when he/she didn't take into account that his/her method would have each person shaking hands twice with everyone else. A demonstration by the 8 children in a circle would soon show him/her the fallacy in this reasoning.

4. R[illegible] e problem and its solution

[illegible] children have solved the problem, the discussion time should focus on having the children introspect.

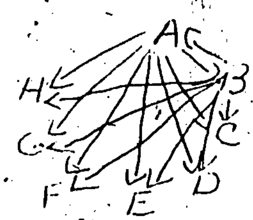
Teacher to 1st student: "You and your group used an experiment. Will you show the class how you solved the problem?"

1st child: "Jean shakes hands with 7 people; Paul shakes hands with 6; Mary shakes hands with 5; Phyllis with 4; Peter with 3; Levi with 2 and Juan with 1.

7 + 6 + 5 + 4 + 3 + 2 + 1 = 28 handshakes."

Teacher to 1st student: "Using an experiment is a good way to solve a problem sometimes. Did everyone follow the solution?"

2nd student: "Hey, that's the same as our drawing! There are 7 lines from A, 6 from B, 5 from C, 4 from D, 3 from E, 2 from F and 1 from G. We only drew lines from A and B but we could see the pattern. We get 28 handshakes, also!"



Teacher to 2nd student: "A diagram is one of the most helpful ways of displaying a problem and often it leads directly to a solution. Here you used the start of a diagram and noted a pattern. I agree that a completed diagram would have been confusing with so many lines.

3rd student: "We come up with 28 handshakes, too - but we like our method best!"

	Jim	Jane	John	Joan	Bob	Beth	Bill	Barb
shakes hands with	Jane John Joan Bob Beth Bill Barb	John Joan Bob Beth Bill Barb	Joan Bob Beth Bill Barb	Bob Beth Bill Barb	Beth Bill Barb	Bill Barb	Barb	
	7	+ 6		4	3	+ 2	+	= 28

Teacher to 3rd student: "Your solution uses an <u>organized listing</u> strategy. Do you see that, here, every handshake is listed in an orderly fashion?"

4th student: "I think we have discovered something. When we used a table to find a pattern we discovered that we could tell the number of handshakes for other parties, too. You see, with each new person the number of handshakes is increased by a number 1 less than the new number of people. That makes sense because if a new guest came and he was the 7th person he would have to shake hands with the other 6."

People	2	3	4	5	6	7	8	9	...
Handshakes	1	3	6	10	15	21	28	36	
New Handshakes		2	3	4	5	6	7	8	

Teacher to 4th student: "That's nice. Can anyone in class tell me how many handshakes there would be for 10 people?...for 20 people?...Can we find a <u>general rule</u> for any number of people?"

5th student: "What's wrong with my reasoning? I get 56 handshakes. Since each person must shake hands with 7 others, there must be 8 x 7 handshakes!"

Teacher: "Who can tell what might be incorrect about that reasoning? It sounds good to me..."

...And so the discussion continues. At some point in the problem-solving instruction the teacher should present a list of strategies. Such a list serves several purposes including helping children consider a variety of attacks on the problem.

The teacher should extend the problem by asking children questions similar to those which follow. By providing extensions of the problem[illegible] children to work o[illegible]ol or at home, children have a chance to use their own strategies, or those of another or to find a generalized solution.

"How many handshakes would there be in your family?"

"How many handshakes would there be in our classroom?"

"How many handshakes would there be if all the boys shook hands just with all the girls?"

In choosing process problems for problem-solving instruction, care should be taken to choose those problems which lend themselves to solution using a variety of strategies, require little formal mathematics, and are interesting to the children. Process problems have an appeal to children; they enjoy solving them. At the same time it is important to use the standard textbook problem to teach problem-solving procedures. In other words, don't miss any opportunity, especially those which arise from classroom, school or community events. Although this article has not specifically addressed the issue of the role of the affect in problem solving, attention to this aspect is important. Helping children become motivated and willing to solve problems is an important prerequisite a teacher must consider.

Among the specific techniques that have been successfully used by classroom teachers in teaching problem solving is the bulletin board - the "Problem of the Week". Using this technique, a new problem is placed on the bulletin board once a week. A discussion follows, as previously described, emphasizing step 1 or "understanding the problem". The children work on the problem until a preassigned day. Often the children involve their parents or friends outside school looking for strategies that might be different or unique. At the preassigned time, discussions as described in the examples are held. This activity together with an emphasis on problem-solving procedures when using the standard textbook problems can provide the basis for a sound curriculum in problem solving.

Teaching problem solving may be a problem. But tackling the challenge of this important facet of elementary school mathematics can be enjoyable and rewarding. The strongest motivation for successful problem solving is success - seeing and hearing children eagerly attack and solve problems. There is little question that children enjoy this activity. Try it. You'll all like it!